

Bell Ringer - Factoring and Complete the Square

Solve each

$$y = 2x^2 + 26x + 60$$

$$y = 2x^2 + 9x + 7$$

$$y = 4x^2 - 100$$

Complete the square

$$y = x^2 + 8x + 4$$

$$y = 1/2x^2 - 4x - 8$$

$$y = 4x^2 + 16x + 36$$

Solving Quadratic Equations without FactoringRecall: To solve by factoring,

- (1) collect all terms on one side of equal sign
- (2) factor the expression
- (3) use $(a)(b) = 0$ to state $a = 0$ or $b = 0$

Consider this example:

$$\begin{array}{ccc} x^2 - 12x + 32 = 0 & & \\ (x - 8)(x - 4) = 0 & & \\ \swarrow \quad \quad \searrow & \text{or} & \swarrow \quad \quad \searrow \\ x - 8 = 0 & & x - 4 = 0 \\ x = 8 & & x = 4 \end{array}$$

4.9 (6.4)-Solving Quadratic Equations without Factoring

Vertex form can also be very useful for solving a quadratic equation.

find roots

(a) Write $y = x^2 - 12x + 32$ in vertex form

$$\begin{aligned} y &= x^2 - 12x + 32 \\ &= x^2 - 12x + 36 - 36 + 32 \\ &= (x-6)^2 - 4 \\ \text{Vertex } (6, -4) \end{aligned}$$

b) Solve for $y = 0$

$$\begin{aligned} y &= (x-6)^2 - 4 \\ 0 &= (x-6)^2 - 4 \\ \sqrt{4} &= \sqrt{(x-6)^2} \\ \pm 2 &= x-6 \\ 6 \pm 2 &= x \\ \swarrow \text{ or } \searrow \\ x_1 &= 8 \quad x_2 = 4 \end{aligned}$$

In some cases, one may be simpler than the other.

Ex.2 Write in factored & vertex form, then choose which to use for solving.

(a) $x^2 + 3x - 4 = 0$

$$\begin{aligned} (x+4)(x-1) &= 0 \\ \swarrow \text{ or } \searrow \\ x &= -4 \quad x = 1 \\ \text{AOS: } \frac{-4+1}{2} \\ x &= -1.5 \end{aligned}$$

$$\begin{aligned} \text{Sub } x &= -1.5 \text{ to check} \\ &= (-1.5+4)(-1.5-1) \\ &= (2.5)(-2.5) \\ &= -6.25 \end{aligned}$$

\therefore factored form is $y = (x+4)(x-1)$ &
Vertex form is $y = (x+1.5)^2 - 6.25$

Ex.2 Write in factored & vertex form, then choose which to use for solving.

(b) $x^2 - 9 = 7$

$$\begin{aligned} x^2 - 9 - 7 &= 0 \\ x^2 - 16 &= 0 \\ \sqrt{x^2} &= \sqrt{16} \\ x &= \pm 4 \end{aligned}$$

Vertex Form

$$y = x^2 - 16$$

Factored Form

$$y = (x - 4)(x + 4)$$

If factoring is not possible use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

which is derived from completing the square (p.337-338).

Note: To use the quadratic formula, the equation must be in standard form, $ax^2 + bx + c = 0$.

The ' \pm ' symbol means there are two solutions.

$$x = \frac{-b \oplus \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-b \ominus \sqrt{b^2 - 4ac}}{2a}$$

Ex.3 Solve using the quadratic formula.

a) $x^2 - 4x - 3 = 0$
 $\begin{matrix} a & b & c \end{matrix}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{28}}{2}$$

$$\rightarrow x_1 = \frac{4 + \sqrt{28}}{2} \doteq 4.65$$

$$\rightarrow x_2 = \frac{4 - \sqrt{28}}{2} \doteq -0.65$$

b) $x^2 - 2x - 5 = 0$

$$\begin{matrix} x_1 \doteq 3.45 \\ x_2 \doteq -1.45 \end{matrix}$$

Assigned Work:

p.343 # 1ad, 3, 4bdf, 5ace, 9ad, 10d, 14, 19*